Free Surface Lattice-Boltzmann Fluid Simulations with and without Level Sets

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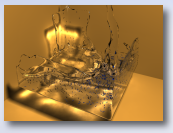
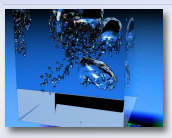
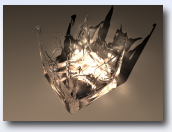
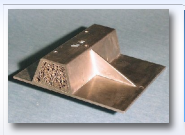
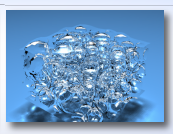
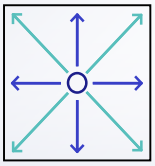




Applications of Free Surface Simulations:
- Computer Graphics (Animation and Visualization of Physical Phenomena)
- Optimization of Production Processes (e.g. Foaming or Casting)

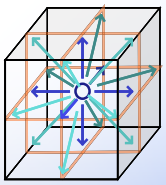




Lattice Boltzmann Method Properties

- A Form of Cellular Automaton (developed from Lattice Gas methods)
- Regular Grid with a fixed number of discrete Velocites
- Timestep and Cellsize are normalized to simplify Calculations
- Optimized Implementation and Parallelization possible


D2Q9

Distribution Functions: Fraction of Particles moving along one of the Grid Velocities.
For 2D a Model with 9 Velocities and DFs is usually used, for 3D a Model with 19 Velocities and DFs.

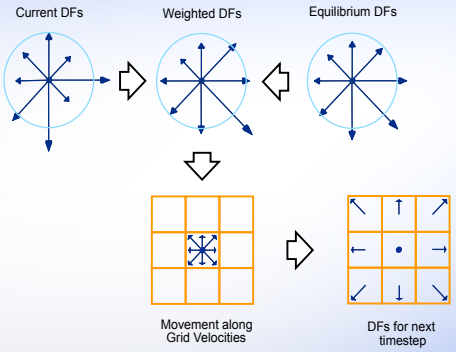

D3Q19

The basic Lattice Boltzmann Algorithm:

During one Timestep:

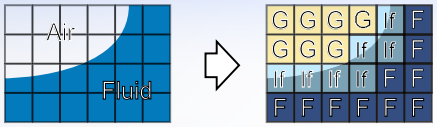
1) **Collide**: Collisions during Particle movement are calculated

2) **Stream**: Movement of Particles during one Timestep along their respective velocites



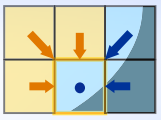
Free Surfaces for Lattice Boltzmann Simulations:

Additional Cell Types are introduced:



- G: Gas: No Computations necessary
- If: Interface: DFs from Gas Region are reconstructed
- F: Fluid: Standard Lattice-Boltzmann Algorithm

Free Surface Boundary Conditions:
Reconstruct incoming DFs for Interface Cells from Gas Pressure and Surface Velocity.

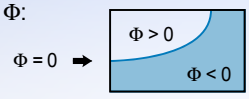


- Reconstructed DF from Gas Region (can also include Surface Tension or higher Gas Pressure)
- DF streamed normally from neighboring Cell

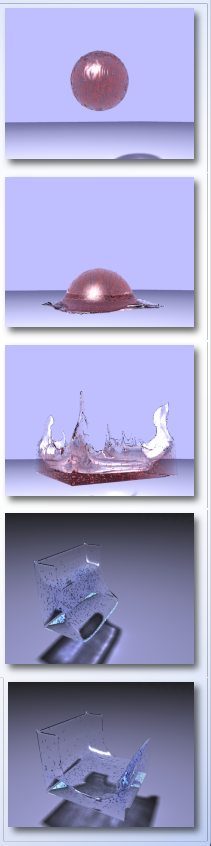
Surface Tracking Algorithms

Level Sets

- Surface Representation as Level Set of the implicit Function Φ:



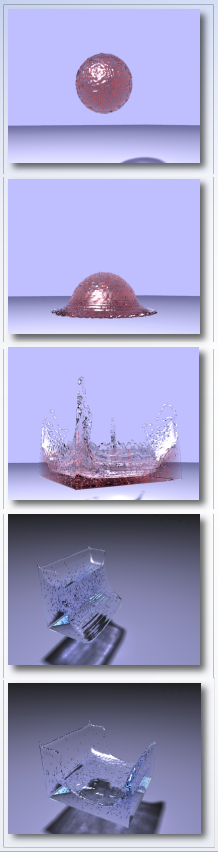
- The Values of Φ are the signed Distance of the Cell Center of a Lattice-Boltzmann Cell to the nearest Point on the Interface
- Φ is advected with the Velocity Field given by the Lattice-Boltzmann Simulation.
- As the Level Set has to be defined in both the Gas and Fluid Regions, the Velocities from the Fluid have to be extrapolated into the Gas Region with the Fast Marching Method.



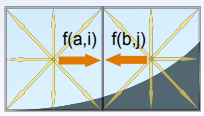
Conclusions:

- Smooth Surface Representation
- Simple Curvature Calculation
- Expensive Velocity Extrapolation
- Mass Conversvation requires Extensions

Mass Tracking



- Mass exchange is calculated for adjacent Interface-Interface and Interface-Fluid Cells:



Change of Mass in Cell a:
$\Delta M(a) = f(b,j) - f(a,i)$
Change of Mass in Cell b:
$\Delta M(a) = f(a,i) - f(b,j)$

- Cell Types need to be changed when Cells fill (Mass >= Density) or empty (Mass <= 0)
- Occurring excess Mass needs to be redistributed to neighboring Interface Cells to guarantee Mass Conservation.

Conclusions:

- Simple & Efficient Implementation
- Mass Conservation guaranteed
- Surface Tracking Artifacts
- Expensive Curvature Calculation